5. Exercises

Exercice 8.1 Determine the derivative functions of the following polynomial functions, defined on $\mathbb{R}$ by : a) $f(x) = 6x^4 - 3x^3 + 2x^2 - 1$
b) $g(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 4x + 2$
c) $h(x) = \frac{3x^3 - 4x^2 + 5x - 1}{5}$

Exercice 8.2 1) Why is the function f defined on the interval $I =]0; +\infty[$ by : $f(x) = \frac{1}{4}x^3 - \frac{3}{2}x^2 + \sqrt{x}$ differentiable on I ?

2) Compute, for all $x \in I$, $f'(x)$.

Exercice 8.3 Compute $f'(x)$ while specifying on which interval(s) your calculation is valid.
a) $f(x) = (2x - 1)(5x + 8)$
b) $f(x) = (\sqrt{x} + 1)^2$
c) $f(x) = (x^2 - x)\sqrt{x}$

Exercice 8.4 Compute $f'(x)$ while specifying on which interval(s) your calculation is valid.

a) $f(x) = -\frac{4}{x^3}$

b) $f(x) = \frac{1-2x}{x-2}$

c) $f(x) = \frac{2-x^2}{2+x^2}$

d) $f(x) = \frac{2x^2}{1-x}$

Exercice 8.5 f is the function defined on $\mathbb{R}$ by :

$$f(x) = \frac{3x}{x^2 + 1}$$

1) Prove that f is differentiable on $\mathbb{R}$. Compute $f'(x)$.
2) Determine an equation of the tangent to $\mathscr{C}$, the representative curve of f, at the point of abscissa a, where a is any number.

Exercice 8.6 f and g are two functions defined on $\mathbb{R} - \{2\}$ by : $f(x) = \frac{4x+1}{x-2}$ and $g(x) = \frac{9}{x-2}$.
1.a) Prove that f and g are differentiable on $]-\infty; 2[$ and on $]2; +\infty[$.
1.b) Compute $f'(x)$ and $g'(x)$.
What do you notice ?
2) Compute $f(x) - g(x)$.
Then justify the remark from question 1.